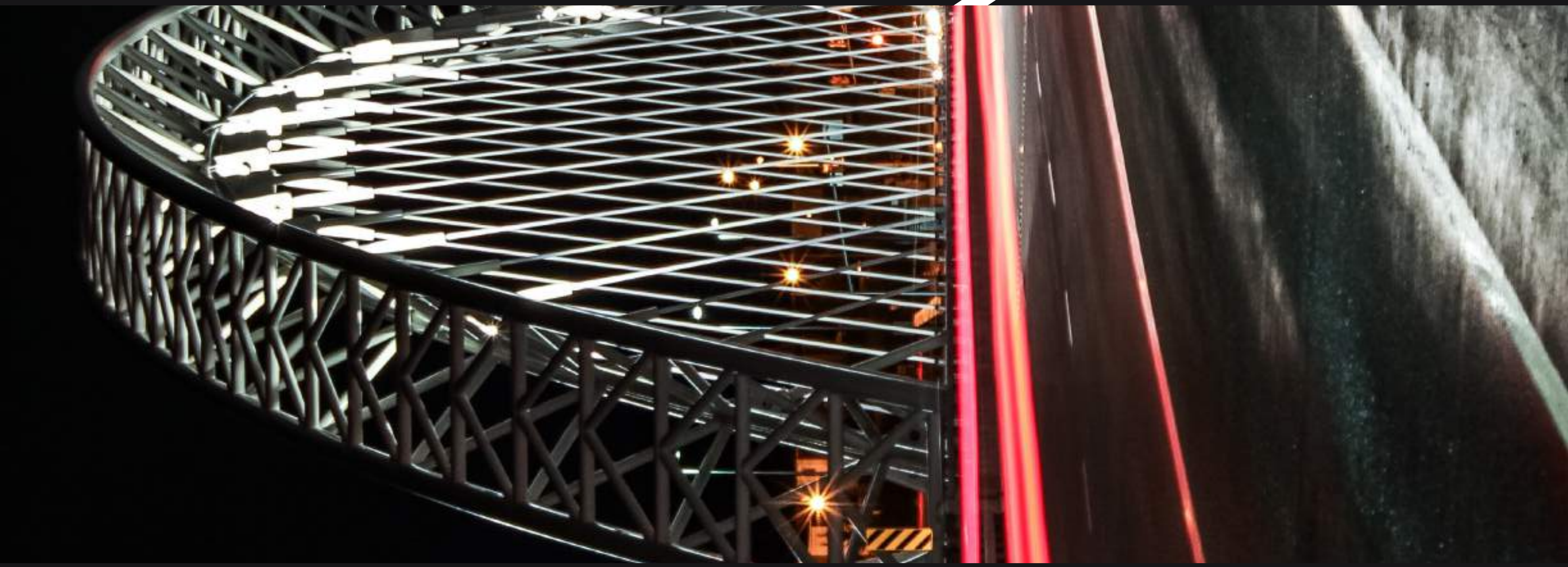

Global Option Markets
Accuracy Validation

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Options Market

The size of option markets has reached astronomical levels. At the end of 2015, the notional amount outstanding of Exchange-traded options and OTC market options are more than 38 trillion and 52 trillion respectively, with total notional amount outstanding being 91 trillions of dollars. (Source: Bank for International Settlement). A significant proportion of exchange-traded and over-the-counter options are American style, offering a flexible early exercise opportunity to options holder.

No closed-form exact solutions are available to calculate the price or sensitivity/Greeks of American options with the exception of a few special cases of American options on no-dividend stocks with positive risk-free rates. Therefore, many numerical techniques and approximations have been developed and used in practice for pricing American options.

Different Pricing Methods

Binomial/Trinomial Trees or Finite Differences

One category of these techniques is based on binomial/trinomial trees (see Cox, Ross & Rubinstein (1979)) or finite differences (see Brennan & Schwartz (1977)). These methods and their later modifications (Chen & Joshi (2012); see also Hull (2014) for a survey) are still widely used and have even served as a benchmark for comparison. Tree methods with 10,000 nodes have been accepted as the true price for American options. However, these methods are extremely time-consuming so they are not very practical. Another serious problem of these methods is that greeks (derivatives by input parameters) can be computed only as ratios of finite differences and therefore cannot be trusted.

One can argue this matters less with the increasing power of modern computers and servers. Nevertheless, in risk application hundreds or thousands of American options are to be repriced in hundreds (historical simulations) or thousands (Monte Carlo simulations) of scenarios and the time consumption can be very important, even when using modern computers and distributed calculation techniques. Due to problems with greeks it also becomes impossible to provide accurate hedging.

Quadratic Approximations

Another popular category is quadratic approximations including MacMillan (1986) and Barone-Adesi & Whaley (1987) methods. Using the Black-Scholes-Merton (1973) classical partial differential equation as a starting pointing, and after making a few simplifying assumptions, they obtained an analytical formulae for the pricing of American puts and calls. These methods use iterative procedures. Methods like Newton-Raphson should be implemented to find a ‘critical price’ for the underlying as part of the pricing procedure. Ju & Zhong (1999) proposed a modification of the Barone-Adesi and Whaley/MacMillan Methods, improving their quality for the intermediate maturity options.

Analytical Methods

In the absence of any closed form formula for American options, a reliable analytical method is obviously highly desirable. First, an analytical method will be very efficient computationally. Second, such a method will be extremely accurate without scarifying computation speed. This is where OQNT come in with such an analytical method. Our new technique to approach American option pricing is accurate, efficient and reliable. In the following content, we will compare OQNT’s pricing results in detail with three other commonly used pricing methods – trees with 150 nodes, quadratic approximation by MacMillan and Barone-Ades (MBAW) and modified quadratic approximation by Ju and Zhong's (JZ).

Numerical Comparison Framework

We use a numerical framework similar to that in Alex Braunstein (2008).

True Price and Error Calculation

The analysis of pricing accuracy, requires a benchmark. Typically, tree with 10,000 steps is considered the “true" value for comparisons. Due to computational time the 10,000 steps is too burdensome in test environments, we instead use a 1000 nodes trinomial tree for comparison to select 10 scenarios where OQNT’s method generates the maximal error. Then, we compared these 10 worst results with the other 3 methods – tree with 150 nodes, MBAW and JZ against a 10,000 nodes trinomial tree. The same type of comparison is done with 10 worst cases for each method.

We use absolute relative error to measure accuracy. Absolute relative error is calculate by taking absolute value of computation price and the benchmark price. Then divide the results by benchmark price.

Numerical Setting

There are six parameters that will influence the option prices. They are stock price S, strike price K, interest rate r, volatility sigma, time to maturity T and dividend rate d. For each of these parameters, we set appropriate value to measure how different methods behave under various situations.

According to Alex Braunstein (2008), the pricing results seems constant across values of different interest rate. As a result, we fix this parameter to 5% (r = 0.05) in our analysis. We also fix strike price K to 100. In the following analysis, we will take ratio of strike to spot into account, so the value of strike price is a non-issue in our analysis.

Time to maturity is an important variable that will affect the prices. For time to maturity T, we varied it with different increments in various time intervals. For interval 0 to 1, the increment is 1/12. For interval 1 to 10, the increment is 0.25.and for interval 10 to 30, the increment is 1.

We varied dividend rate from 0 to 2*r in increments of 0.01.

Followed the settings above, there are 748 pairs in total for time to maturity and dividend rate (T, d). For every fixed pair (T, d), we varied stock price S from 50 to 200 with an increment of 10. We also varied volatility from 0.2 to 2 with an increment of 0.05 for put options and from 0.2 to 3 with an increment of 0.05 for call options.

Comparison Results

Showing all the pricing results is possible but it is really unclear and unnecessary. In order to demonstrate the accuracy of the OQNT method, we pick up the situation which OQNT generates the worst result. For every (T, d) pair, we found the maximal absolute relative error against a 1,000 trinomial tree and took the 10 (T, d) pairs with the highest maximal absolute relative error, and graphed them against a 10k trinomial tree.

For each of these pairs we display a 3D chart, with Z-axis being the absolute relative error, X-axis being volatility, and Y-axis being moneyness, the ratio of spot price to strike price (S/K). **Z-axis** is from the origin up, **Y-axis** is from the origin right, **X-axis** is from the origin pointing towards the reader.

We noticed that some surfaces gave us an absolute relative error of 103.01, or 10301% error. Upon further investigation, we realized that this only happens when the tree and our algorithm prices the option close to 0 (eg. When OQNT price is 1.322e-13 while Tree price 1.343e-15, the error became

98.77 or 9877%). We omit such cases in our analysis, and we applied this to all methods in our comparisons.

The OQNT method for pricing was tested at standard precision, and our error was never above 0.01% when compared to the 10k node trinomial tree, in other words, we were never off by a penny! From our extensive testing, we also noticed that the 150 node trinomial tree (commonly used by exchanges as a “fast and still accurate” shortcut) is really inaccurate when the volatility goes up (ie. trading volume is large). Another thing to note is that even when the 150 node tree gives 1-2% error, that is major, the price could be off by cents or even dollars. This difference is in turn multiplied by the number of shares per contract (at least 100) and by the trade volume (the number of contracts).

We show 4 groups of graphs as follow:

Case 1 – OQNT Worst Case:

The first group of charts is a scenario that our OQNT method generates the maximal error. This happens for put options with the (T, d) pair as (0.0833, 0.04).

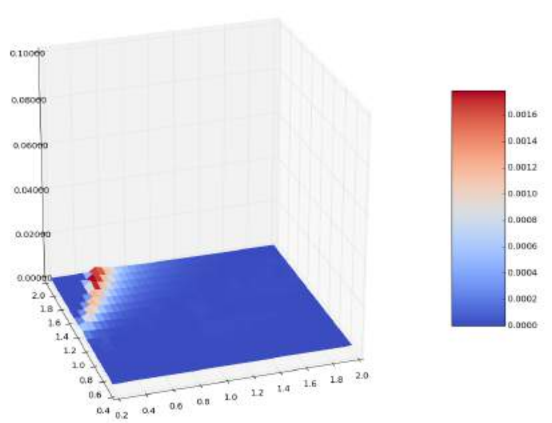

Chart 1 OQNT Method Case 1

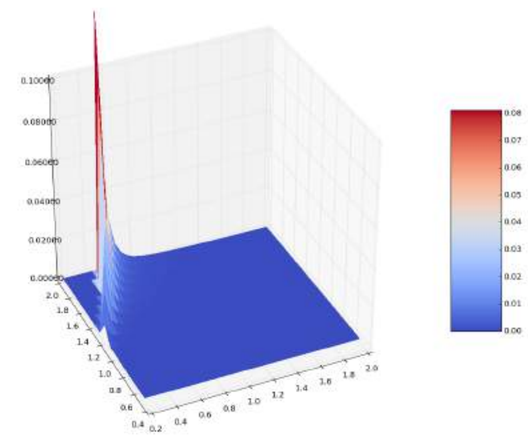

Chart 2 Ju-Zhong Case 1

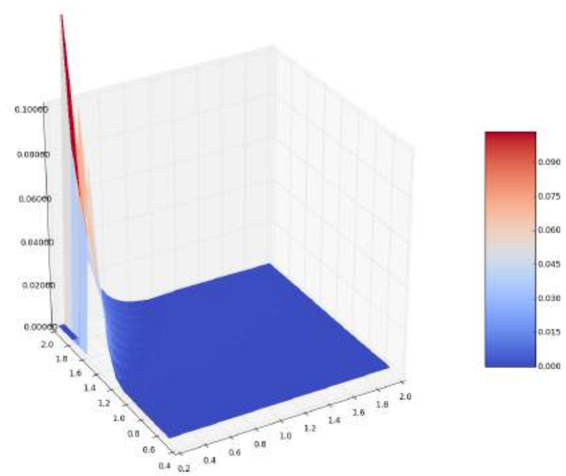

Chart 3 150 Nodes Tree Case 1

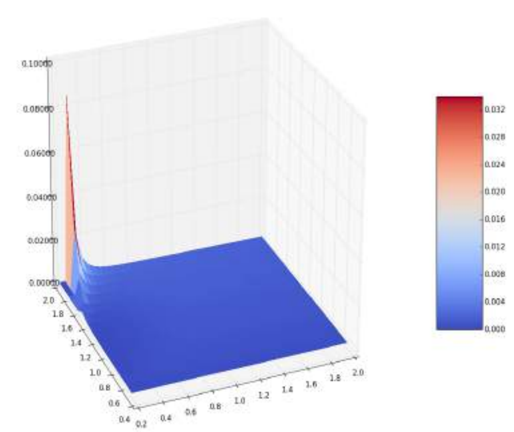

Chart 4 MBAW Case 1

Even though this chart is the worst situation that OQNT got, the maximal absolute relative error is below 0.002. OQNT is really super accurate.

Under this scenario, the other three methods drive results much more inaccurate than OQNT, with worst error up above 0.08 — 40 times worse than OQNT. The worst results happen when the put options are out of the money with low volatility.

From the charts we can see that trinomial tree with 150 nodes generate more error results that other methods.

A reminder: Z axises' scale are the same for 4 graphs. The color of the 3D plot shows the relative high or low error in a graph. The intervals of the color are not the same for these four graphs.

Case 2 – Ju-Zhong Worst Case:

The second group of charts shows that Ju-Zhong method generates the maximal error. It happens for put options with (10.0, 0.1) as their (T, d) pair. That is to say, Ju-Zhong is not good to price put options with long time to maturity and low dividend rate.

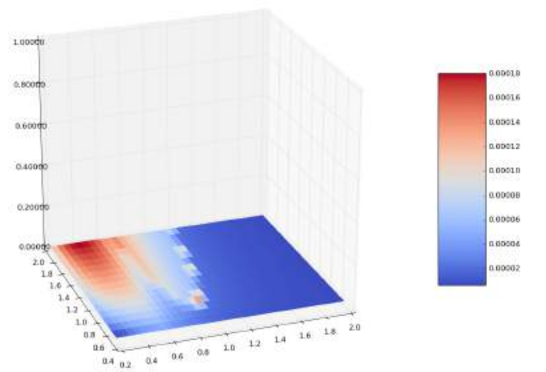

Chart 5 OQNT Method Case 2

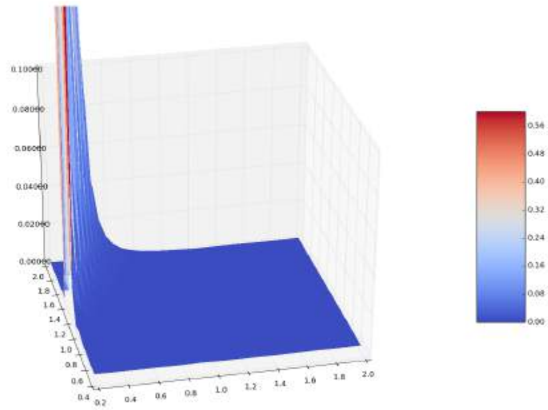

Chart 6 Ju-Zhong Case 2

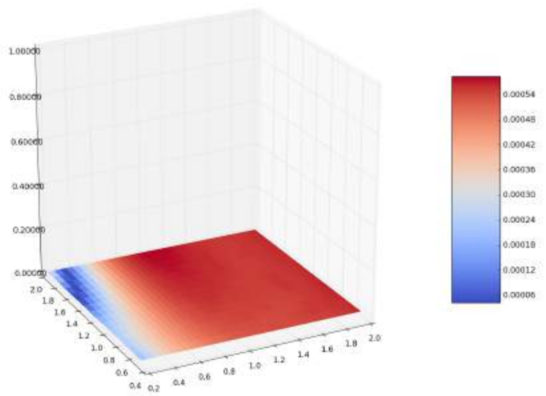

Chart 7 150 Nodes Tree Case 2

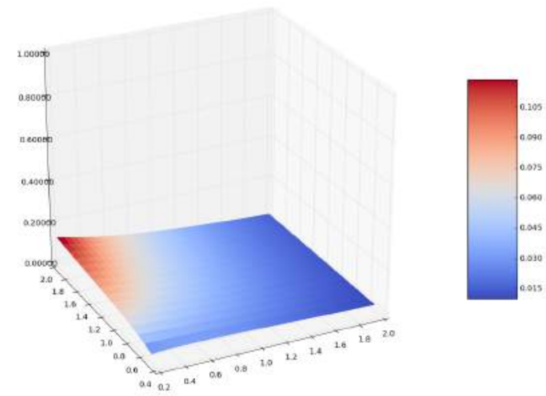

Chart 8 MBAW Case 2

Again, OQNT shows great accuracy in this case. The maximal error of OQNT is around 0.00018, which is really small. The maximal error is above 0.56 for Ju-Zhong, 3000 times more than that of OQNT. Again, the worst results for Ju-Zhong happen when the put options are out of the money with low volatility. In general Ju-Zhong can behave uncontrollably, blow up and even result in a negative early exercise premium [please refer to Boussygine, Rodriguez]. The MBAW's maximal error is above 0.105, 580 times more than OQNT method.

Trinomial tree with 150 nodes is good compared to Ju-Zhong and MBAW. However, it is worse than OQNT especially when volatility goes up. For long time to maturity option, volatility is an essential element since it greatly influences the extrinsic value of an option.

Case 3 – MBAW Worst Case:

The next group of charts is a scenario that MBAW method generates the maximal error. This happens for a put option with the (T, d) pair as (9.25, 0.1). We omit Ju-Zhong since its error goes off scale.

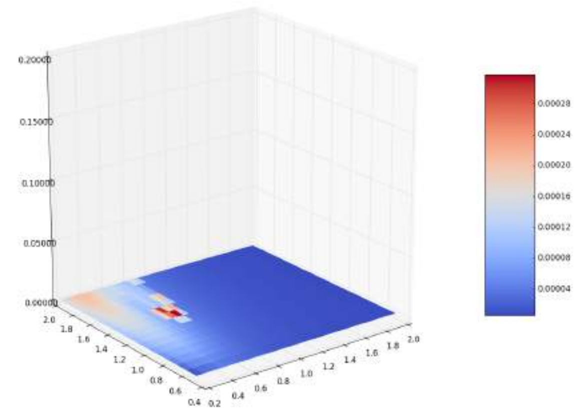

Chart 9 OQNT Method Case 3

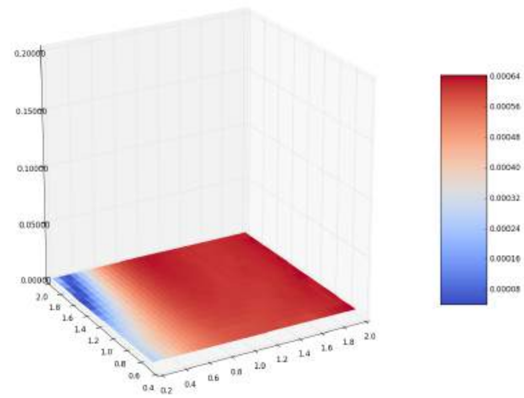

Chart 10 150 Nodes Tree Case 3

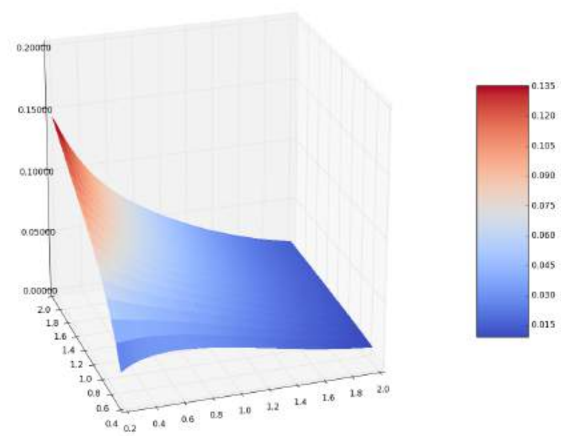

Chart 11 MBAW Case 3

Most of OQNT error is under 0.00008 and even the maximal error in this case is only 0.0003, while most of the MBAW error is around 0.07 and the maximal absolute relative error go up to 0.13.

Trinomial tree with 150 nodes has error around 0.00064

Case 4 – 150 Nodes Tree Worst Case:

The last group of charts is a scenario that 150 Nodes Tree method generates the maximal error. This happens for a call option with (T, d) pair (0.75, 0.02). We omit Ju-Zhong and MBAW, because their error goes off scale.

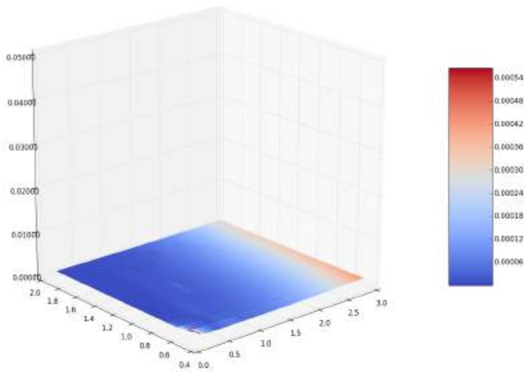

Chart 12 OQNT Method Case 4

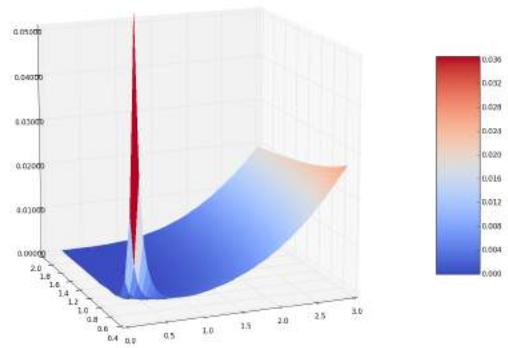

Chart 13 150 Nodes Tree Case 4

In this case, the trinomial tree with 150 nodes has an error range from 0 to 0.036. From the chart we can clearly see that the pricing accuracy decreases as the volatility goes up. Not only in this case, also in most cases that the tree method with 150 nodes start doing worse when the volatility increases. Typically, it is great to trade options when the volatility is high since it means there are great trading volume and there is opportunity to gain profit. That tree method behaves badly under high volatility will greatly affect the option trading results.

OQNT is still doing great in this case with most error under 0.00015. All charts above demonstrate that OQNT provides stable, consistent and most important – accurate pricing results!